November 2012 subject reports

MATHEMATICAL STUDIES

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–16	17–31	32–44	45–57	58–70	71–82	83–100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–4	5–6	7–8	9–11	12–14	15–16	17–20

The range and suitability of the work submitted

There appeared to be an overall drop in standards this session with many candidates writing short, rushed projects that did not appear to satisfy the 20 hours of class work and a similar amount of time for homework. There were also candidates who submitted a partial project of one or two pages in order to avoid being disqualified from obtaining a diploma.

Most candidates chose to write a statistical project. Other types of project using modelling, optimization, probability, trigonometry, sequences or finance were few and far between. The topics for the projects were mainly suitable and most had titles. The clarity of the statements of task was variable as was the details of the plan. It is important for the candidate to write a clear plan explaining what they are going to do, what mathematical processes they are going to use in the project and give reasons as to why they are using these processes. This will help them to focus and will prevent them from including any irrelevant processes in their project.

Nearly all projects contained data which varied from a few pieces to hundreds of pieces. It should be noted that having a lot of data does not necessarily mean that it is quality data. However, there were several examples of projects with good quality data this session. Some candidates did not include their raw data. This then makes it impossible for the moderator to check if the tables are set up correctly or if the mathematical processes are accurate. Also, some candidates forgot to attach a copy of their questionnaire or survey. When using a random sample of data, the candidates should give an explanation of their method of choosing the “random” sample.

The simple mathematical processes were often done using technology without any explanation. The candidate should give an example of how to find a mean or show how to calculate the angles at the centre of a pie chart. A few candidates did not include any simple processes and jumped right into a chi-squared test and, as a result, their chi-squared test was counted as their first simple process and they did not score well in criterion C.

The main errors in the sophisticated processes were, as always, in the chi-squared test (no null hypothesis stated, raw data or percentages instead of frequencies in the table of observed values, too many entries less than 5 in the table of expected values) and regression (drawing or calculating the regression line when the correlation coefficient was not moderate or strong). Much use was made of technology with results occurring often without working, interpretation or justification. The teacher should encourage some calculations, as it is difficult for the moderator to verify that the candidates knew what they were doing. Most projects had at least one interpretation that was consistent with the analysis. Many candidates are now able to gain one mark for validity but very few are able to achieve full marks for this criterion. Projects generally had some structure but not always appropriate notation and terminology.

The guidance given to the candidates varied from school to school as did the quality of the teacher's comments on the 5/PJCS form. It is important for the teacher to write a comment against each of the assessment criteria, explaining why they awarded the marks, as this is helpful during the moderation process. Teachers should also write comments on the project and check the accuracy of the mathematics.

Candidate performance against each criterion

Criterion A

Many candidates managed to gain two marks for this criterion. Those who did not, usually did not have any clear plan to describe what they were going to do or their project did not have a title. Teachers should stress the importance of writing a clear statement of the task and a clear and detailed plan of how they are going to achieve this. This focuses the candidate and usually results in a project that is clear and follows a logical order.

Criterion B

Many candidates collected data that was appropriate for their project but this was not always sufficient to perform the mathematical processes laid out in their task nor was it always good quality data. Few candidates describe their sampling method and this is something that teachers could focus more on. Candidates who are using data from the internet or other secondary sources must also remember to identify the source in their bibliography. All raw data must be included in the project in order for the moderator to check the accuracy of tables and mathematics. Data that is too simple has a knock on effect for the whole project as it limits the mathematical processes that can be applied, the interpretations and the communication.

Criterion C

In many of the projects the simple mathematics was done using technology. These processes such as mean, median, pie charts could all have been demonstrated by hand, showing the moderator that the candidate knew what they were doing. Some candidates missed out any simple mathematical processes and only did a chi-squared test or line of regression. When no simple processes are present then the first sophisticated process is counted as simple. It is important for the student to realize this. As mentioned above there are still many errors in the chi-squared test and candidates are still drawing lines of regression on diagrams where there is little or no correlation present. This makes the process irrelevant and lowers the mark awarded for this criterion.

Criterion D

The project flows better if partial conclusions are made after every mathematical process and then an overall conclusion given at the end. Most candidates managed to give at least one interpretation that was consistent with their analysis but fewer could produce thorough explanations of their calculations, often due to the fact that the project was too simple. Some students attempted to justify their results based on their own personal beliefs rather than the mathematics that they had performed.

Criterion E

Candidates are now commenting more on their data collection, their results and giving suggestions for extensions or improvement. Few are able to comment successfully on the validity of the mathematical processes that they have used throughout their project.

Criterion F

Many projects have a reasonable structure but, due to errors in notation and terminology, only receive 1 mark for this criterion. The most common errors are: * for multiplication, ^ for power of, x^2 for the chi-squared symbol, E for 10 to the power of …, mixing up the correlation coefficient and the coefficient of determination.

Criterion G

The majority of the teachers award this appropriately. Some schools abuse it and give full marks to all their students irrespective of the quality of the project.

Recommendations and guidance for the teaching of future candidates

Teachers can help their candidates in many ways:

- Make sure that they read the Subject Report.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Give them examples of “good” projects so that they know what is expected of them.
- Peer assessment is a wonderful tool. Let the students moderate each other’s projects.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the students the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.

- Encourage students to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Emphasize the importance of meeting deadlines.
- Inform their students about sampling techniques.
- Remind them to include all raw data either in an appendix or as part of the task.
- Show their students how to use Equation editor or MathType.
- Remind them of the importance of including simple mathematical processes in their projects.
- Check the calculations in each project.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–28	29–43	44–54	55–66	67–77	78–90

The areas of the programme and examination which appeared difficult for the candidates

- Total surface area of a solid cylinder
- The converse of a logical statement
- Equation of the axis of symmetry of a quadratic function
- Using mid-class intervals to find an estimate of the mean
- Using right-angled trigonometry in a 3D shape
- Periodic functions
- Compound interest
- Calculus (Local maxima and minima)

The areas of the programme and examination in which candidates appeared well prepared

- Stem and leaf diagrams
- Arithmetic and geometric sequences
- Standard form and percentage error
- Currency conversions
- Gradients and equations of straight lines
- Using right-angled trigonometry in 2 dimensions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Stem and leaf diagrams

There was some evidence to suggest that this was not well taught by some centres as box and whisker diagrams or simply an ordered list was seen on a number of scripts. In part (a), missing a key for the stem and leaf diagram proved elusive to a significant minority of candidates. Irrespective of incorrect diagrams, virtually all candidates identified the median correctly in part (b). 17 proved to be a popular (but incorrect) answer to the last part. Clearly, Q_3 had been incorrectly identified by a significant number of candidates as 177.

Question 2: Arithmetic sequence

Much good work was seen in parts (a) and (b) showing that many centres had well prepared their students for questions on arithmetic sequences. In part (c) however there was poor use of $S_n = \frac{n}{2}\{\text{first term} + \text{last term}\}$ with the incorrect calculation $\frac{250}{2}\{3+250\}$ seen on a significant number of scripts.

Question 3: Mean, standard form and percentage error

Another well answered question with candidates showing a good understanding of standard form and many correct answers were seen in parts (a) and (b). Whilst the formula is given for percentage error, there were still a minority of candidates who divided by 105 rather than the required value of 104.9.

Question 4: Mensuration

There were two common errors identified on a number of scripts in this question. In part (a), the majority of candidates correctly wrote down $\pi r^2 = 8$ for method but a significant number of candidates then went on to write down $r = 2.55$ (forgetting to square root). Such candidates were able to recover in subsequent parts of the question with the allowable follow through marks, but a method mark and final accuracy mark was lost on many scripts in part (c) with incomplete methods shown for the total surface area of the cylinder. Leaving off the addition of either one or both end surfaces of the cylinder resulted in the final two marks being lost.

Question 5: Cumulative Frequency

Candidates who drew vertical or horizontal lines at correct positions on the graph were able to pick up the three method marks for this question and, for parts (a) and (b), a range of answers were accepted. The most common error on this question seemed to be in part (a) where the vertical line was drawn at 49% leading to a value outside the acceptable range of 190–200. Candidates are expected to read values off a continuous cumulative frequency curve at the given critical values (in this instance at 50%). In part (b), a common incorrect answer seen was 350 which was simply the number of candidates who were awarded less than a grade C rather than those with a grade C or higher. On a minority of scripts, an answer of 90 reflected the candidate's misinterpretation of the requirement of the question as 'grade C only'. In part (c), a numerical answer of 60 was required, with 'grade C' on its own losing this last mark.

There was a mistake in the Spanish translation of question 5, which was discovered prior to marking. The principal examiner was informed and this unfortunate situation was addressed during the marking and awarding in order to ensure no candidate was disadvantaged.

Question 6: Geometric sequence

In part (a), 1.1025 proved to be a popular, but erroneous, answer. Similarly to question 4, such candidates failed to find a square root. Whilst this accuracy mark was lost for such candidates, much good work was seen in this question reflecting how well drilled the majority of candidates were in both arithmetic and geometric sequence techniques.

Question 7: Currency conversions

This question was generally well done with the majority of candidates correctly using the currency conversions (multiplying in part (a) and dividing in part (b)) and, in most cases, working out the commission correctly in part (b). On a few scripts, candidates failed to round either one or both of their answers correctly to two decimal places and, as a consequence, lost a mark.

Question 8: Gradients, perpendiculars and equation of a line in two dimensions

The majority of candidates were able to write down the gradient of the straight line in part (a) but a correct answer for the gradient of the perpendicular proved to be more elusive in part (b)(i). Many however recovered in the remainder of the question as they were able to find the equation of a line using their gradient and the coordinates of a point on the line but, in some case, did not always show clear working. A significant minority of candidates, who attempted to substitute (4, 5) into the equation $y = mx + c$, incorrectly identified the value of c as 5.

Question 9: Logic

In part (a) occasionally *'if…then…'* was not seen but generally this was well done. Part (b) was also well done despite the dearth of previous testing of the *exclusive or* statement. Finding the converse of a statement in part (c) proved to be difficult for a significant number of candidates and incorrect answers of the form $q \Rightarrow \neg p$ were more frequently seen than the correct answer. Such incorrect answers lost two marks.

Question 10: Quadratic function

This question proved to be quite a discriminator with a significant number of candidates achieving, at most, one or two marks in part (a). Part (b) was tested in the May 2012 series of examinations but the same errors were prevalent here as they were then. A number of candidates simply wrote the equation of the axis of symmetry in terms of *y* rather than *x* or just wrote down a numerical value rather than an equation. Expressions for the required range in part (c) fared little better with again much confusion between the variables *x* and *y*. A strict inequality was required at the turning point and a mark was lost where this was not indicated. Alternative forms for the range such as $(-\infty\ ,\ 5.125]$ were, of course, accepted.

Question 11: Grouped frequency distribution

Part (a) was generally well done but, in part (b), writing down the mid-interval value of a class proved difficult for some candidates and many incorrect answers of 7.5 were seen. Popular, but erroneous answers, seen in part (c) were 15.5 and 16. These seemed to be as a result of adding their mid-class values together and dividing by 7 rather than the total of the frequencies. There was much confusion over the meaning of the phrase *'at least'* in part (d) and, as a consequence, there were as many wrong answers of $117(30+26+29+32)$ seen as there were correct answers.

Question 12: Right-angled trigonometry

This seemed to be a good discriminatory question enabling the majority of candidates to at least score well on part (a). Challenges arose for candidates who were then required to see the problem in three dimensions for the remainder of the question. Indeed, a significant number of candidates correctly identified the required lengths for part (b) and, provided they used Pythagoras correctly, were able to pick up the marks in this part of the question. However, in part (c), invariably the wrong triangle was chosen with triangles BFM and BAF proving to be the most popular, but incorrect triangles, chosen.

Question 13: Periodic function

In part (a), find *a* and *c* proved to be done reasonably well. However, many candidates simply wrote down 120 as their answer for *b*. In part (b), many answers were seen which were outside the required range for x. Although there were no method marks for this part of the question, the candidates incorrect values found in part (a) were followed through in part (b). Except for the ablest candidates, this question was not done well.

Question 14: Simple and compound interest

In part (a), 10 proved to be as popular an answer as the required answer, as the phrase *'investment doubled'* seemed to indicate to some candidates that this meant the interest doubled to 24 000. Candidates who showed working could gain 2 marks out of 3 marks for this misinterpretation. In part (b), writing down any substituted form of the compound interest formula led to a significant number of candidates scoring at least 1 mark for method here. Often, however, the formula was incorrectly substituted or was not correctly equated to 45 000 and subsequent marks were then lost.

Question 15: Calculus

Many candidates lost 1 mark in part (a) through not realizing that the derivative of x is 1. As a consequence, $15x^2 - 8x$ proved to be a popular answer. Very few candidates gained the marks in part (b) to find the maximum and minimum point. Although the question indicated to use their answer to part (a), very few candidates set the derivative to zero which would have given them 1 mark. It seemed as if many candidates were trying to use their calculators to find the coordinates but could not find which was the maximum and which was the minimum.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- Critically examine their answers to see whether or not they are sensible in the context of the problem set.
- Show all working to enable method marks to be obtained if answers are incorrect.
- Not cross out their work unless it is to be replaced – crossed out working earns no marks at all.
- Completely cover the syllabus content and, in particular, focus on those areas of the syllabus which provide more challenging questions identified on this paper.
- Spend time on answering past paper questions. Most of the questions on this paper could have been familiar to candidates if they had answered the many past papers that are published.
- Ensure that they are fully conversant with the formulae which appear in the information booklet and where exactly these formulae are to be found in the booklet prior to the examination.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–28	29–39	40–50	51–61	62–72	73–90

The areas of the programme and examination which appeared difficult for the candidates

- Understanding of the standard deviation and its application
- Determining situations in which it is appropriate to use the regression line
- Interpreting set notation
- Converting units (e.g. cm^3 to m^3)
- Interpretation and use of a function when defined with a parameter
- Finding and using the gradient function when defined with a parameter
- Conditional probability
- Finding the equation of a vertical asymptote of a graph
- Sketching a graph (using a GDC to graph a function and copying it on paper from their GDC)
- Drawing a tangent to a curve at a given point

The areas of the programme and examination in which candidates appeared well prepared

- Using GDC to find mean, and standard deviation
- Finding the correlation coefficient and the equation of a regression line
- Applying the cosine and sine rule
- Calculating simple probabilities
- Solving simultaneous equations (using the GDC)
- Carrying out and analysing a chi-squared test
- Using GDC to find *x*-intercepts and maximum/minimum points

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Statistics and use of equation of regression line

The question was for the most part approached by almost all candidates and answered relatively well. The question in part (e) related to the use of the equation of the regression line for predicting, although regularly asked on exams, was still found to be a difficult one by some candidates. Some answers still suggested mathematical thinking and language unaccustomed to drawing conclusions and providing justifications.

Question 2: Venn diagram, sets and probability

The question was moderately well answered. The majority of candidates answered part (a) and at least parts of (b), and (d). Part (c) proved to be difficult, as it required understanding and interpreting set notation. Part (e) was rarely answered in its entirety. Part (f) was answered by many candidates, but most of them offered a partial answer to part (g); a typical response was 36 instead of 37.

Question 3: Trigonometry

The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

Question 4: Probability and Chi-squared test

Part (a) was generally well answered by most of the students, except for part (a)(iv) which called for conditional probability. Most students correctly stated the null hypothesis in part (b), and answered parts (d), (e), (f) and (g). In some responses to part (c) it seemed that the difference between calculation of the expected value and showing that the value is 79 was not clear to the candidates. It is important that teachers explain to their students that in a "***show that"*** *question* they are expected to demonstrate the mathematical reasoning through which the given answer is obtained.

Question 5: Calculus

This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation. Finding the derivative in part (b) proved problematic as well. It seems that the presence of the parameter *b* in the function may have contributed to this.

In part (c) a great number of students substituted $b=5$ in the equation of the function instead of substituting it in the equation of their derivative. Very few students used the GDC to find the equation of the tangent at $x=1$ in part (d). Good use of the GDC was seen in part (e), although some students wrote the *x*-coordinates of the point of intersection and neglected to write the y-coordinate. The sketch in part (f) was, for the most part, not well done. Often the axes labels were missing. Very few tangents to the curve at the correct point were seen. Often the intended tangent lines intersected the curve, which shows that candidates either did not know what a tangent is or did not make sense of the sketch. Good use of the GDC was shown in part (g) for finding the coordinates of the minimum point. Few acceptable answers were given in part (h).

Recommendations and guidance for the teaching of future candidates

- Help your students understand the command terms: Find, Justify, Sketch, Draw, Explain, and Calculate.
- The command term “Show that…” must be clearly explained to future candidates so that they know what such a question requires. The ‘Show that’ command demands that students state both the unrounded and final answers.
- Candidates should be advised to set an appropriate window when using their GDC for graphs. They also need to be reminded that labels and clearly identified scales are essential.
- Draw constant attention to the accuracy of your students’ calculations and answers. Students should be reminded to show and use unrounded answers as much as possible.
- Help your students develop conceptual understanding, and ask them to make sense of their methods and answers.
- Remind students to:
    - read each question carefully.
    - start each question on a new page.
    - show units.